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AN ATMOSPHERIC TURBULENCE MODEL FOR THE DYNAMICALLY AIMED FREE---ETC(U)
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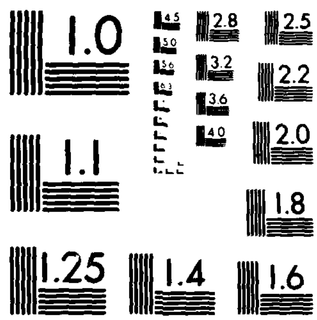

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TECHNICAL REPORT RD–82–1

AN ATMOSPHERIC TURBULENCE MODEL FOR THE DYNAMICALLY AIMED FREE–FLIGHT ROCKET (DAFR) ERROR ANALYSIS

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1 October 1981

U.S. ARMY MISSILE COMMAND
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Previous error analysis of the Dynamically Aimed Free-flight Rocket (DAFR) utilized a linear model for the wind uncertainty. Also, the data used in the model had been filtered.

This effort utilized a first-order Markov process to model the wind longitudinal turbulence uncertainty. The effect of terrain roughness is considered.

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PREFACE

The author wishes to acknowledge the literature search done by Dr. Dorathy A. Stewart, Aerophysics Branch, Research Directorate, US Army Missile Laboratory, US Army Missile Command.

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I. INTRODUCTION

Law of Revelation: "*The Hidden Flaw Never Remains Hidden.*"

Simply stated, the dynamically aimed free-flight rocket (DAFR or DAFFR) concept is to decrease the sensitivity of an artillery rocket to other error sources by increasing wind sensitivity. The wind effect is reduced during firing by round-to-round correction in aiming based upon measurements of the previously fired rounds. The concept has been described in detail by McCorkle and Lilly [1], and some refinements to the error analysis were later made by Gibson.[2]

It was assumed [1,2] that the measurements, z, would be averaged to obtain an unbiased minimum variance estimate, $\hat{x}$,

$$\hat{x} = \frac{1}{N} \sum_{k=1}^{N} z_k \qquad (1)$$

The variance of the estimate is

$$\sigma_{\hat{x}}^2 = \frac{1}{N^2} \sum_{k=1}^{N} \sigma_k^2 \qquad (2)$$

and, assuming the variance, σ_k^2, is constant,

$$\sigma_{\hat{x}}^2 = \frac{\sigma^2}{N} \qquad (3)$$

The variance of the wind is not constant and was modeled as

$$\sigma_{u'}^2 = N^2 \gamma^2 \qquad (4)$$

where N is the number of rounds fired and γ is the sensitivity slope which

[1] William C. McCorkle, Jr., and J. A. Lilly, "An Adjusted Fire Technique for a Highly Accurate Free Flight Rocket Artillery System," US Army Missile Command, Tech. Rpt. RD-74-13, Redstone Arsenal, AL, 25 June 1974 (AD B007539L).

[2] J. D. Gibson, "Error Analysis of a Dynamically Aimed Free Rocket," Texas A&M University, College Station, TX (Unnumbered and undated, not held by DTIC, formerly DDC, or RSIC).

was derived from data in the Tactical Artillery Rocket Environment (TARE) report.[3]

Since

$$\sum_{k=0}^{N} k^2 = \frac{N(N + 1)(2N + 1)}{6} \qquad (5)$$

the resulting variance is

$$\sigma^2_{\hat{x}} = \left(1 + \frac{1}{N}\right)\sigma^2_p + \frac{\sigma^2_\nu}{N} + \left[1 + \frac{(N + 1)(2N + 1)}{6N}\right]\gamma^2 \qquad (6)$$

where σ^2_p is the sum of the squares of all Gaussian white plant noises, and σ^2_ν is the variance of the observation noise. Equation (6) eventually diverges as N becomes large because the linearity assumption, Equation (4), breaks down. Though additional data may not improve an estimate, it should not make the estimate worse.

The TARE report[3] states that "Readings were taken with aerovane type anemometers located at heights of approximately 60 feet and using a smoothing circuit of one (1) minute." The times between rounds previously considered[1,2] were around five to ten seconds. The time delay characteristics previously used were most likely those of the "1 minute time filter" not the wind!

This leaves us with two questions:

- What are the (unfiltered) characteristics of the wind?
- What is their effect on the error analysis?

II. WIND AND TURBULENCE

Thumb's First Postulate: *"It is better to solve a problem with a crude approximation and know the truth, ±10%, than to demand an exact solution and not know the truth at all!"*

The wind model presented below was developed mainly from the results of a literature search on atmospheric turbulence done by Stewart.[4]

---

[3] Tactical Artillery Rocket Environment (TARE) Committee, "Tactical Environment for Large Free Flight Rocket Systems," US Army Ballistic Missile Agency Report, R-S-61-1, Redstone Arsenal, Alabama, 1 June 1961 (AD B952069L).

[4] D. A. Stewart, "A Survey of Atmospheric Turbulence Characteristics," US Army Missile Command, Technical Report RR-81-6, Redstone Arsenal, Alabama, 19 August 1981.

Figure 1 is a plot of the horizontal wind power spectral density.[5,6] The spectral gap between eight hours and ten minutes serves to explain why we may speak of an average wind. To reduce randomness one should operate as far as possible on the other side of the peak due to turbulence.

Assuming[7,4] that the longitudinal wind, u , is the sum of the longitudinal average wind, $\bar{u}$, and the turbulent wind, u' , then

$$u(t) = \bar{u} + u'(t) \tag{7}$$

The turbulent wind is related to the turbulent wind at some previous time by[7,4]

$$u'(t + \tau) = \rho(\tau)\, u'(t) + u''(t) \tag{8}$$

where $\rho(\tau)$ is the autocorrelation coefficient for a time delay, τ , and $u''(t)$ is the random component of the turbulence.

The variance, $\sigma^2_{u''}$, of the random components is defined by the relationship[7,4]

$$\sigma^2_{u''} = \sigma^2_{u'}(1 - \rho^2(\tau)) \tag{9}$$

so that the turbulent energy, $\sigma^2_{u'}$, be conserved with time.

The standard deviation due to turbulence, $\sigma_{u'}$, is related to the average wind, $\bar{u}$, by the approximation[6,4]

$$\sigma_{u'} = \frac{\bar{u}}{\ell n\left(h/z_0\right)} \tag{10}$$

where h is the height and z_0 is the roughness length[8], Figure 2. The

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[5] A. J. McDonald, Wind Loading on Buildings, New York: John Wiley & Sons, 1975.

[6] J. L. Lumley and H. A. Panofsky, The Structure of Atmospheric Turbulence, New York: John Wiley & Sons, 1964.

[7] S. R. Hanna, "Some Statistics of Lagnangian and Eulerian Wind Fluctuations," Journal of Applied Meteorology, Vol 18, April 1979, pp. 518-525.

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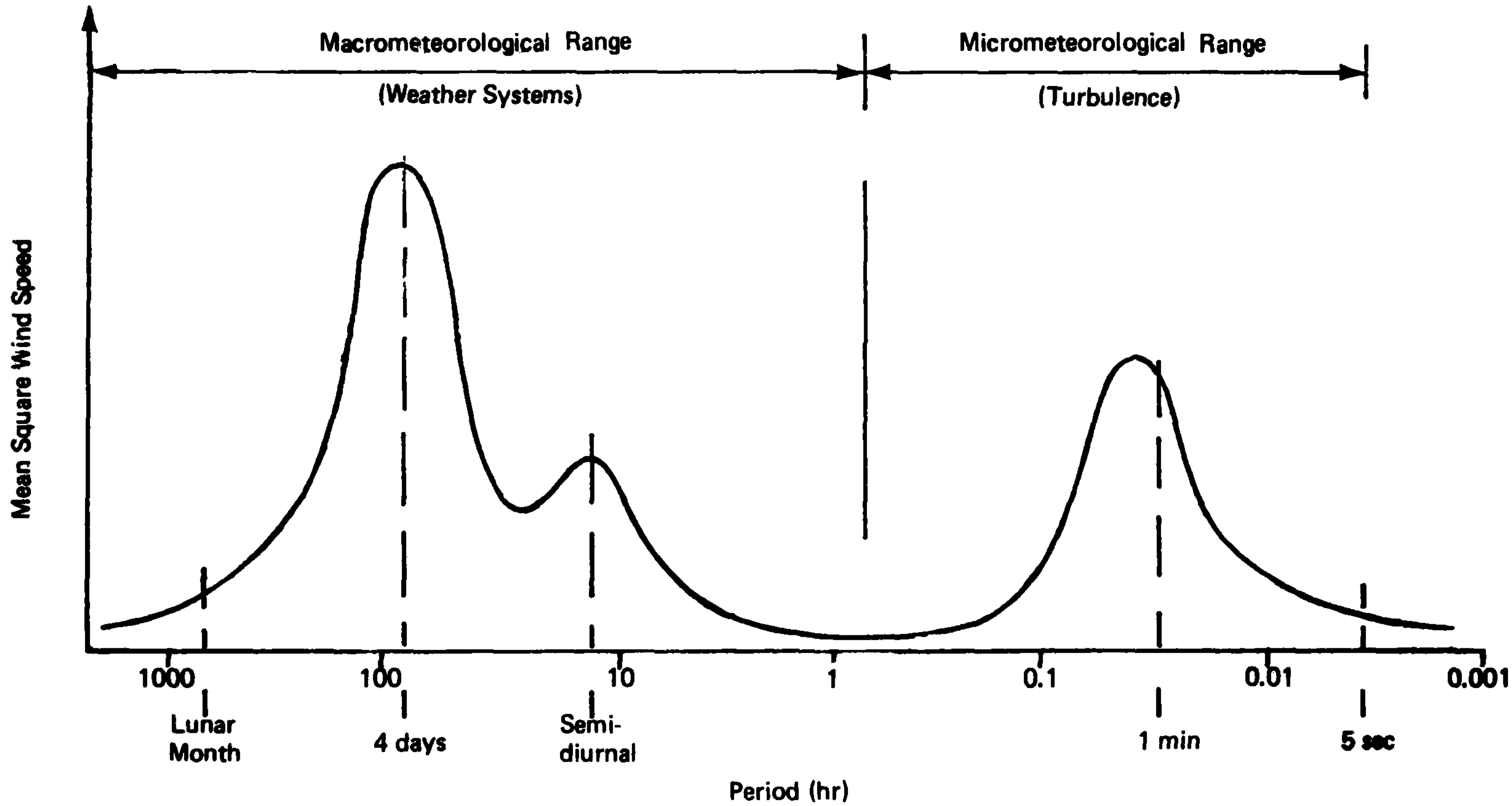

Figure 1. Power spectrum of horizontal wind speed.

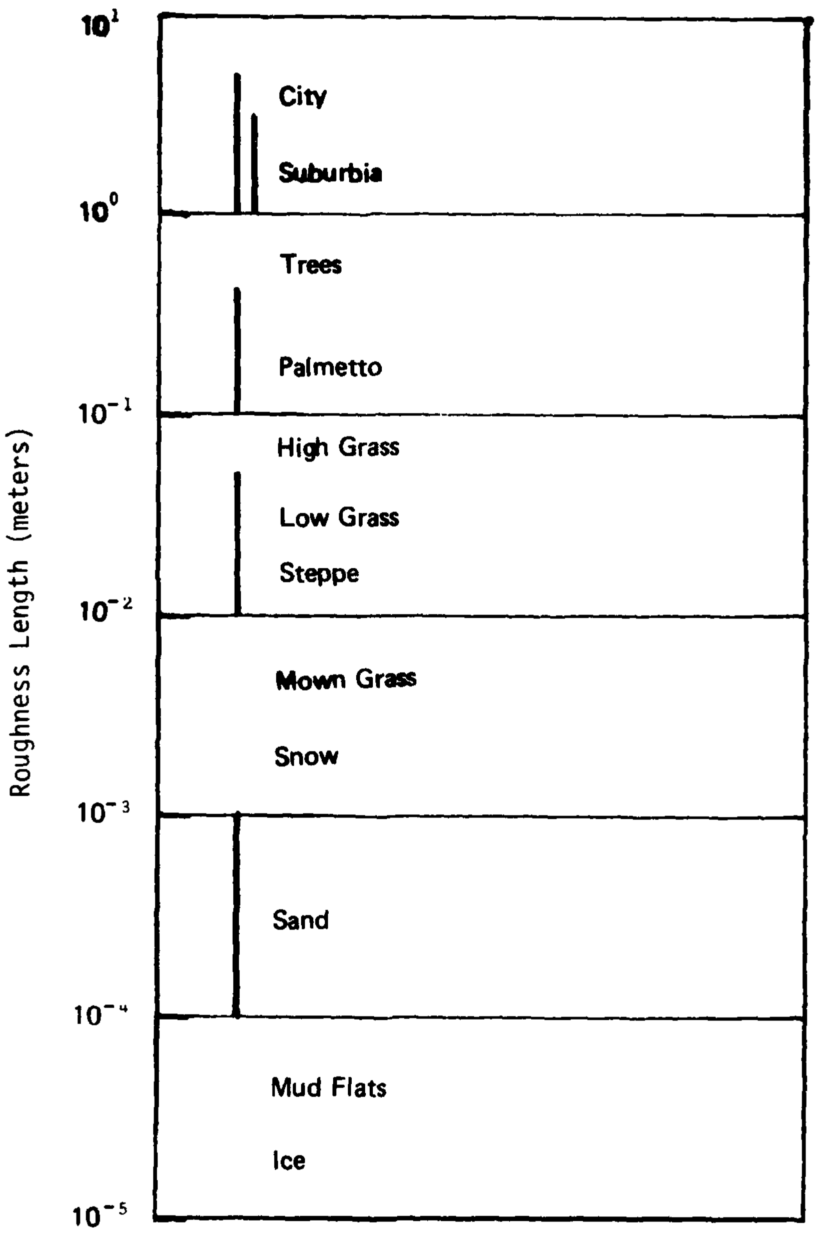

Figure 2. Roughness length for various terrains.

lateral components are approximately[9]

$$\sigma_v = (0.64)\sigma_{u'} \tag{11}$$

and

$$\sigma_w = (0.52)\sigma_{u'} \tag{12}$$

Of course, $\overline{v} = \overline{w} = 0$ by definition. These approximations hold best for a neutral atmosphere, which implies strong winds.[4]

Figure 3 is a plot of the intensity, $\sigma_{u'}/\overline{u}$, against height for various roughness lengths.

The Dryden correlation coefficients[9] are as follow:

- Longitudinal

$$\rho_u(\tau) = e^{-\overline{u}\,\tau/L_u} \tag{13}$$

- Transverse

$$\rho_v(\tau) = e^{-\overline{u}\tau/L_v}(1 - \overline{u}\tau/2L_v) \tag{14}$$

$$\rho_w(\tau) = \rho_v(\tau) \tag{15}$$

where L is the isotropic turbulence integral scale.

In the atmospheric boundary layer[9],

$$L_u = \frac{25\ h^c}{z_o^{0.4}} \tag{16}$$

where

$$c = e^{-0.025(\ell n\ z_o)^2 + 0.17\ \ell n\ z_o - 0.8} \tag{17}$$

[9] W. Frost, B. H. Long, and R. E. Turner, "Engineering Handbook on the Atmospheric Environmental Gudielines for Use in Wind Turbine Generator Development," NASA Technical Paper 1359, December 1978.

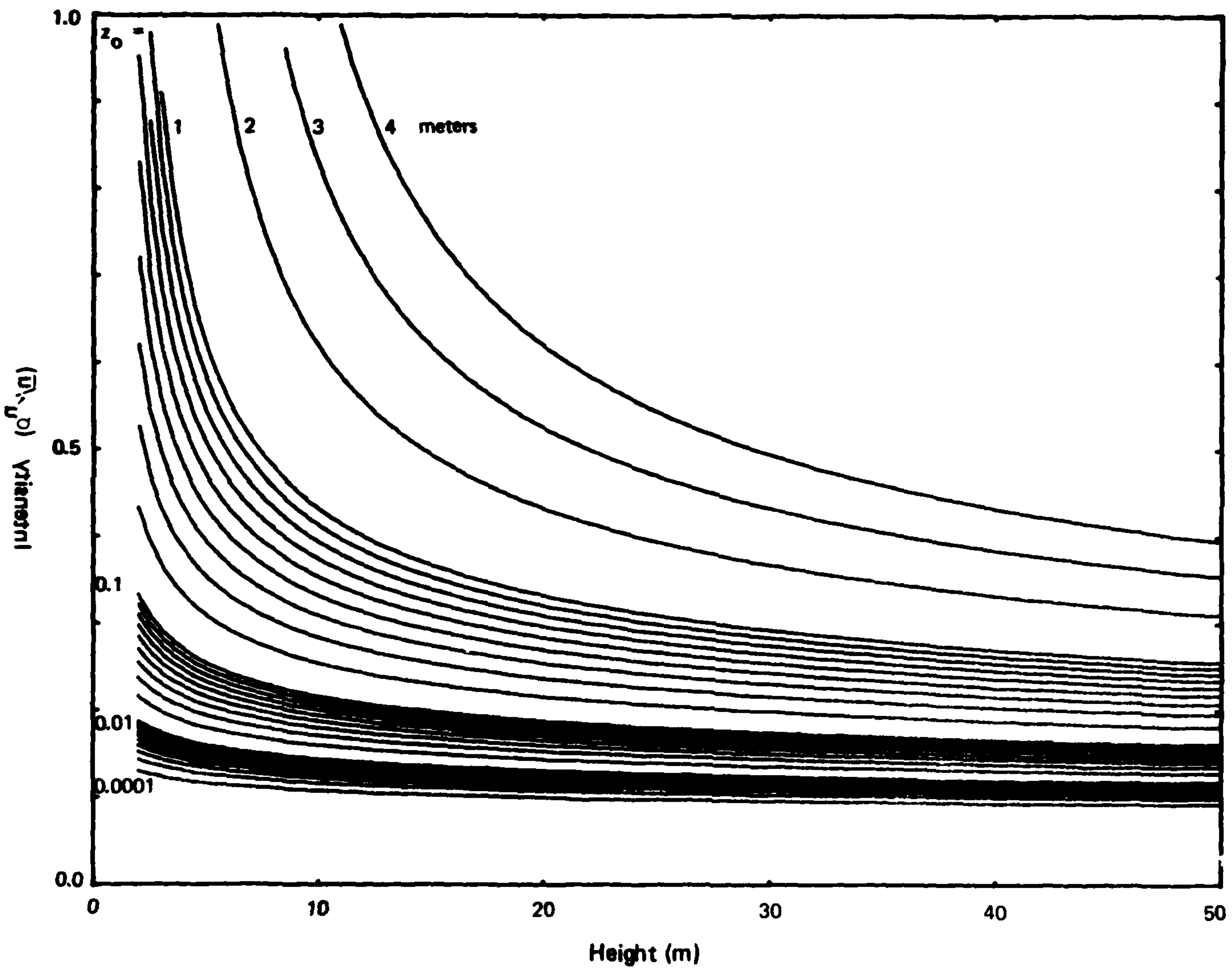

Figure 3. Intensity versus height.

and

$$L_v = L_w = (0.4)h \; , \; h < 250 \text{ meters.} \tag{18}$$

For $h \geq 250$ meters

$$L_u = L_v = L_w = 250 \text{ meters} \tag{19}$$

Figure 4 is a plot of L_u against height for various roughness lengths. To illustrate how the formula/figures may be used, consider a height of 5 meters and a roughness length of 0.1 meter. From Figure 3

$$\sigma_{u'} = (0.25)\, \bar{u} \tag{20}$$

and from Figure 4

$$L_u = 100 \text{ meters} \tag{21}$$

Substituting into Equation (13) and then Equation (9) results in

$$\sigma_{u''} = (0.25)\, \bar{u} \left(1 - e^{-2\bar{u}\tau/100}\right)^{1/2} \tag{22}$$

A plot of Equation (22) is shown in Figure 5. L_v and L_w are so short, Equation (18), that the transverse components are essentially white noise and correction is not feasible, though they would contribute to the error budget.

The above model for the turbulent wind corresponds to a Gauss-Markov process, that is, Gaussian white noise passed through a low pass filter where the order of the process is the order of the filter[10]. This model is only reasonable for the most right-hand portion of Figure 1, but for time delays of less than one minute it is within 20 percent of observed autocorrelation coefficients.[7] An in-depth discussion of the bases for the model will be found in the report by Stewart.[4]

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[10] A. Gelb (ed.), Applied Optimal Estimation, Cambridge: MIT Press, 1974.

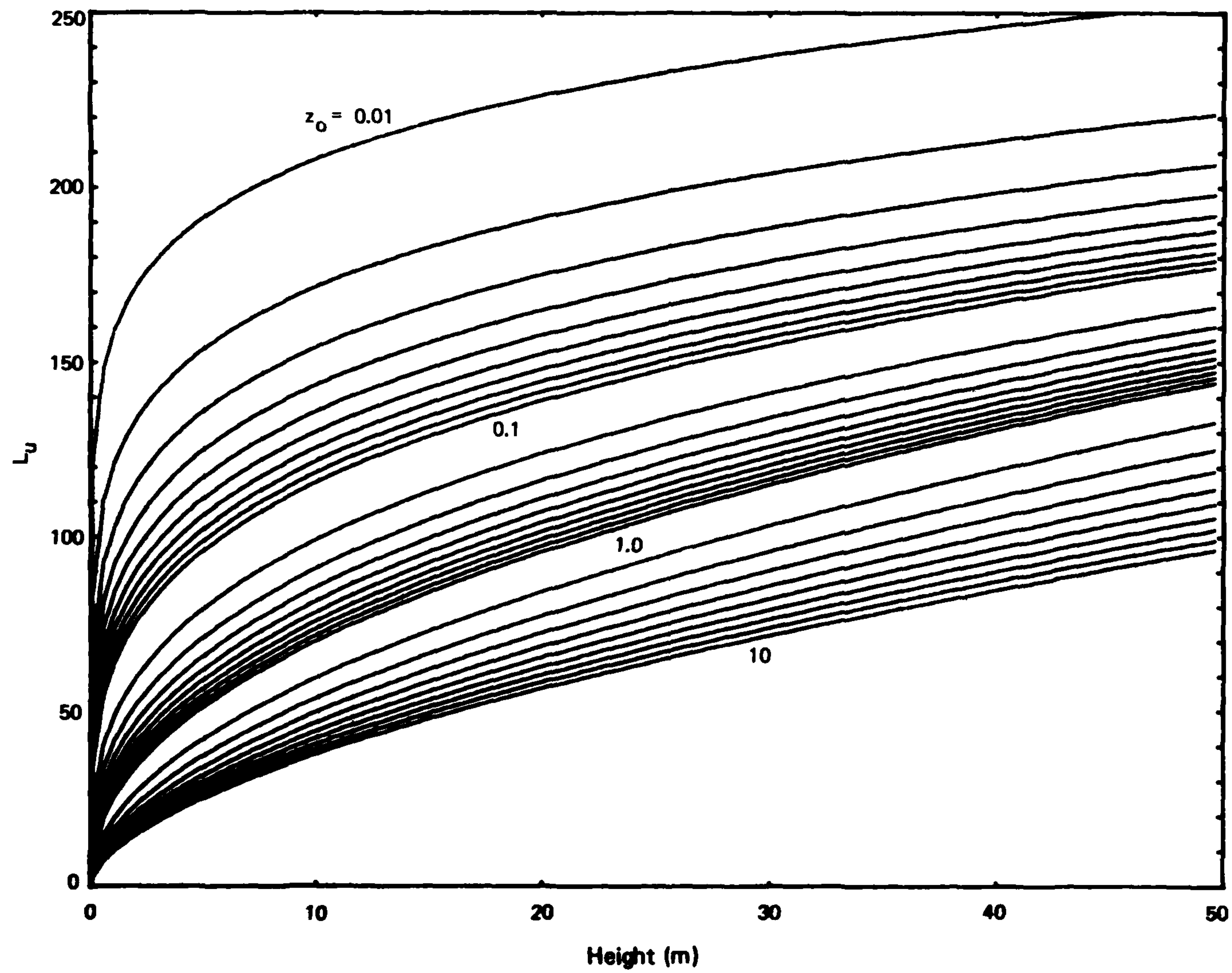

Figure 4. Turbulence integral scale versus height.

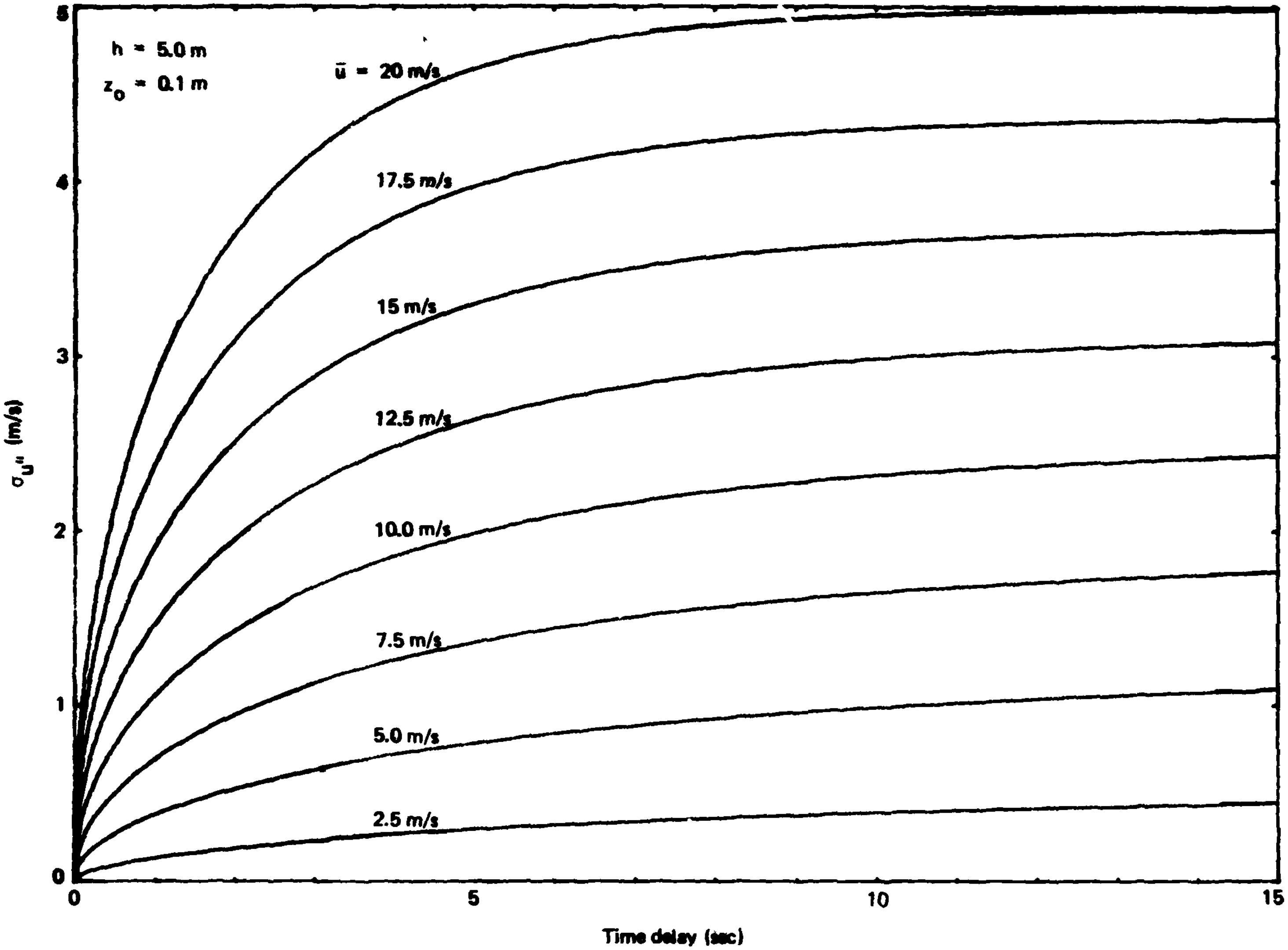

Figure 5. Turbulence standard deviation versus time delay.

III. THE ERROR ANALYSIS — REVISITED

Grossman's Misquote of H. L. Menchen:

"Complex problems have simple, easy to understand wrong answers."

Instead of using

$$\sigma_{u''}^2 = N^2 \gamma^2 \tag{4}$$

use the following in Equation (2):

$$\sigma_{u''}^2 = \sigma_{u'}^2 \left(1 - e^{-2\bar{u}\tau/L_u}\right) . \tag{6}$$

Since

$$\sum_{k=0}^{N} \bar{e}^{\,ak} = \frac{1 - \bar{e}^{\,a(N+1)}}{1 - \bar{e}^{\,a}} \tag{23}$$

for

$$\tau = N T , \tag{24}$$

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{N^2} \sum_{k=1}^{N} \left(\sigma_p^2 + \sigma_\nu^2 + \sigma_{u''}^2\right) \tag{25}$$

becomes

$$\hat{\sigma}_\varepsilon^2 = \frac{\sigma_p^2 + \sigma_\nu^2}{N} + \frac{\sigma_{u'}^2}{N^2} \left[N + 1 - \left(\frac{1-e^{-2(N+1)\bar{u}\,T/L_u}}{1-e^{-2\bar{u}\,T/L_u}}\right)\right] \tag{26}$$

For large $\bar{u}\,T/L_u$, Equation (26) approaches

$$\hat{\sigma}_\varepsilon^2 = \frac{\sigma_p^2 + \sigma_\nu^2 + \sigma_{u'}^2}{N} \tag{27}$$

that is, the Gauss-Markov process "becomes" essentially Gaussian white noise.

Since [1,2]

$$\hat{\sigma}_{x_{N+1}}^{2} = \sigma_p^{2} + \sigma_{u''}^{2} + \hat{\sigma}_{\varepsilon_{N+1}}^{2} \tag{28}$$

substituting Equation (26) yields

$$\hat{\sigma}_{x_{N+1}}^{2} = \left(1 + \frac{1}{N}\right)\sigma_p^{2} + \frac{\sigma_\nu^{2}}{N}$$

$$+ \sigma_{u'}^{2}\left\{\left(1 - e^{-2\bar{u}T/L_u}\right) + \frac{1}{N^2}\left[N+1 - \left(\frac{1-e^{-2(N+1)uT/L_u}}{1-e^{-2\bar{u}T/L_u}}\right)\right]\right\} \tag{29}$$

Plots of Equation (29) are given in Figures 6 through 11. The standard deviations have been arbitrarily expressed in mils; though representative, the values do not reflect an actual system. The quantity, $\bar{u}T/L_u$, requires some explanation. For a given turbulence integral scale, for example,

$$L_u = 100 \text{ meters}$$

and an average wind

$$\bar{u} = 5 \text{ meters per second}$$

for

$$\frac{\bar{u}\,T}{L_u} = 0.1$$

it follows that the time between rounds must be

$$T = 2 \text{ seconds}$$

For a small precision error, Figure 6, the improvements from decreasing the time between rounds, T , are dramatic. For a large precision error, Figure 11, there is not much improvement at all since that error dominates.

These figures may be used for other integral scales, L_u , and average winds, $\bar{u}$, but the time between rounds, T , would need to be recomputed.

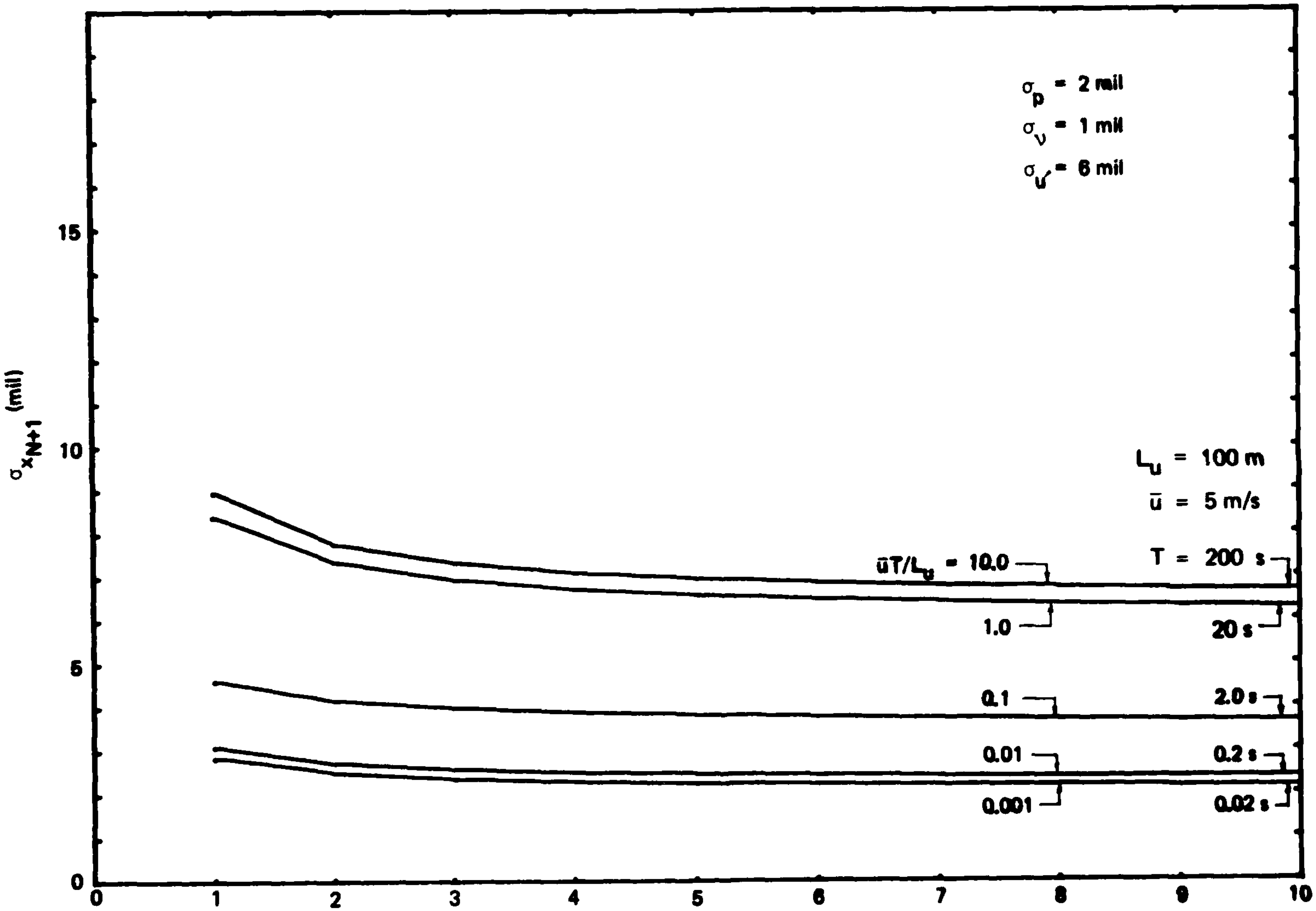

Figure 6. Number of rounds already fired.

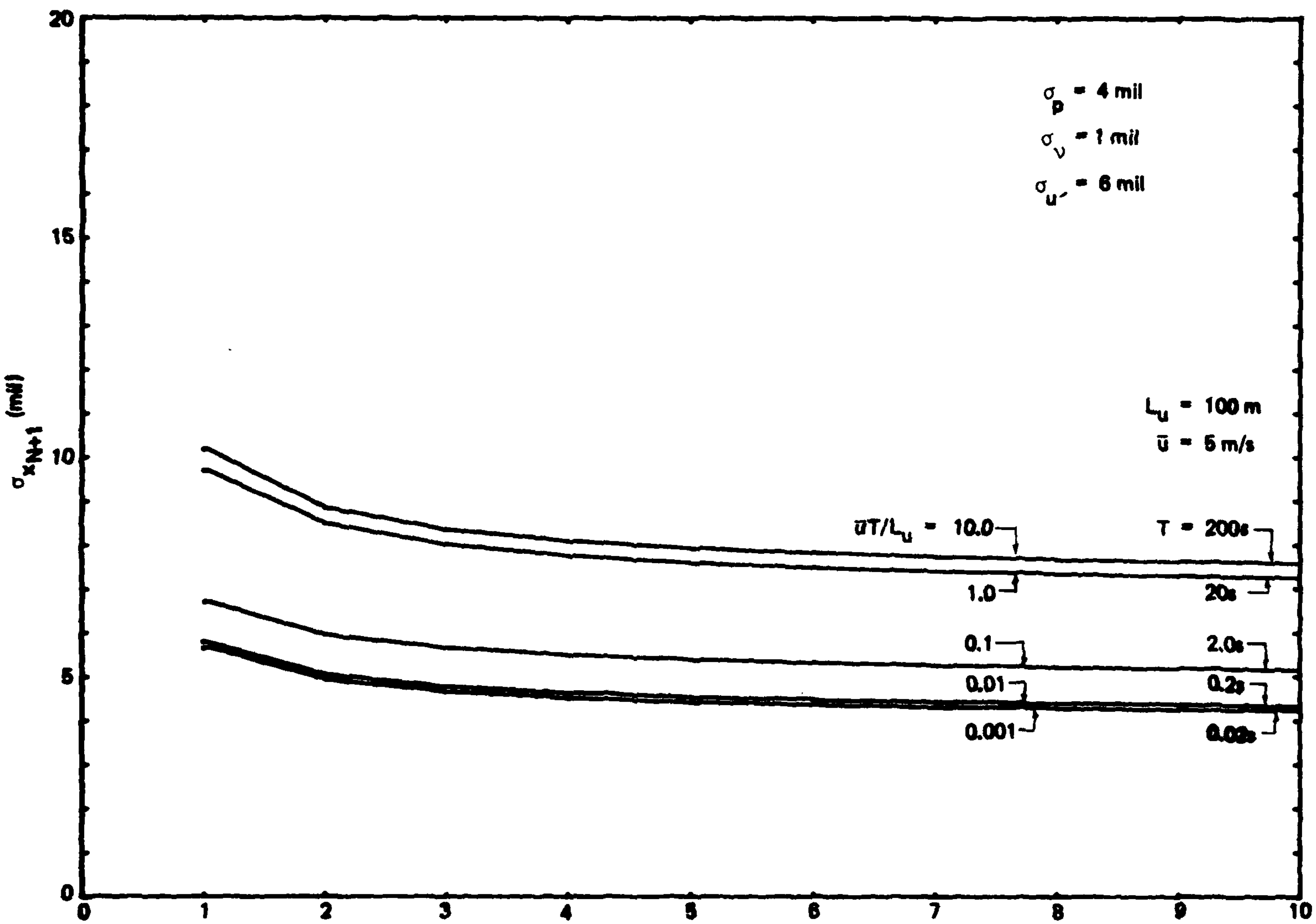

Figure 7. Number of rounds already fired.

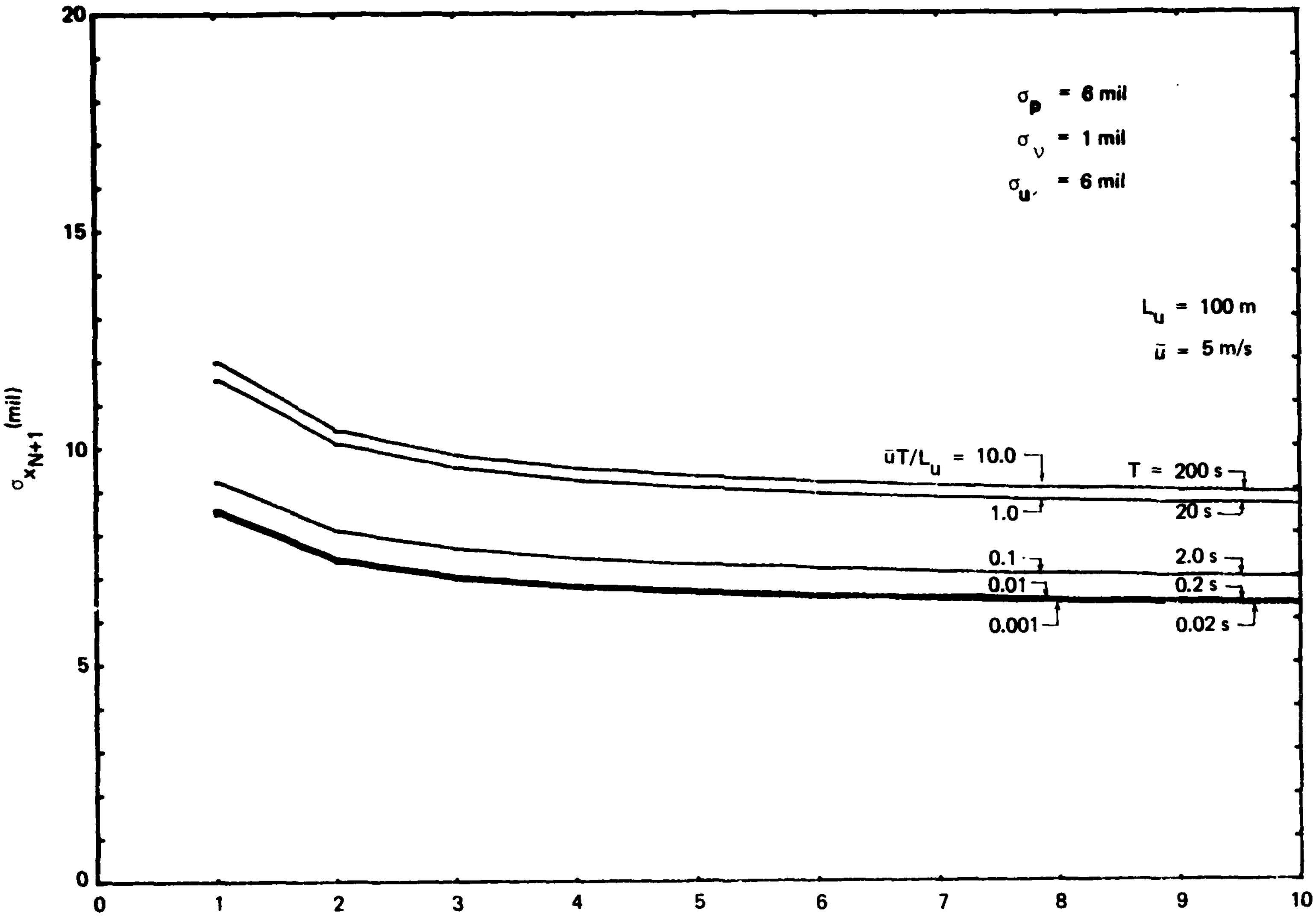

Figure 8. Number of rounds already fired.

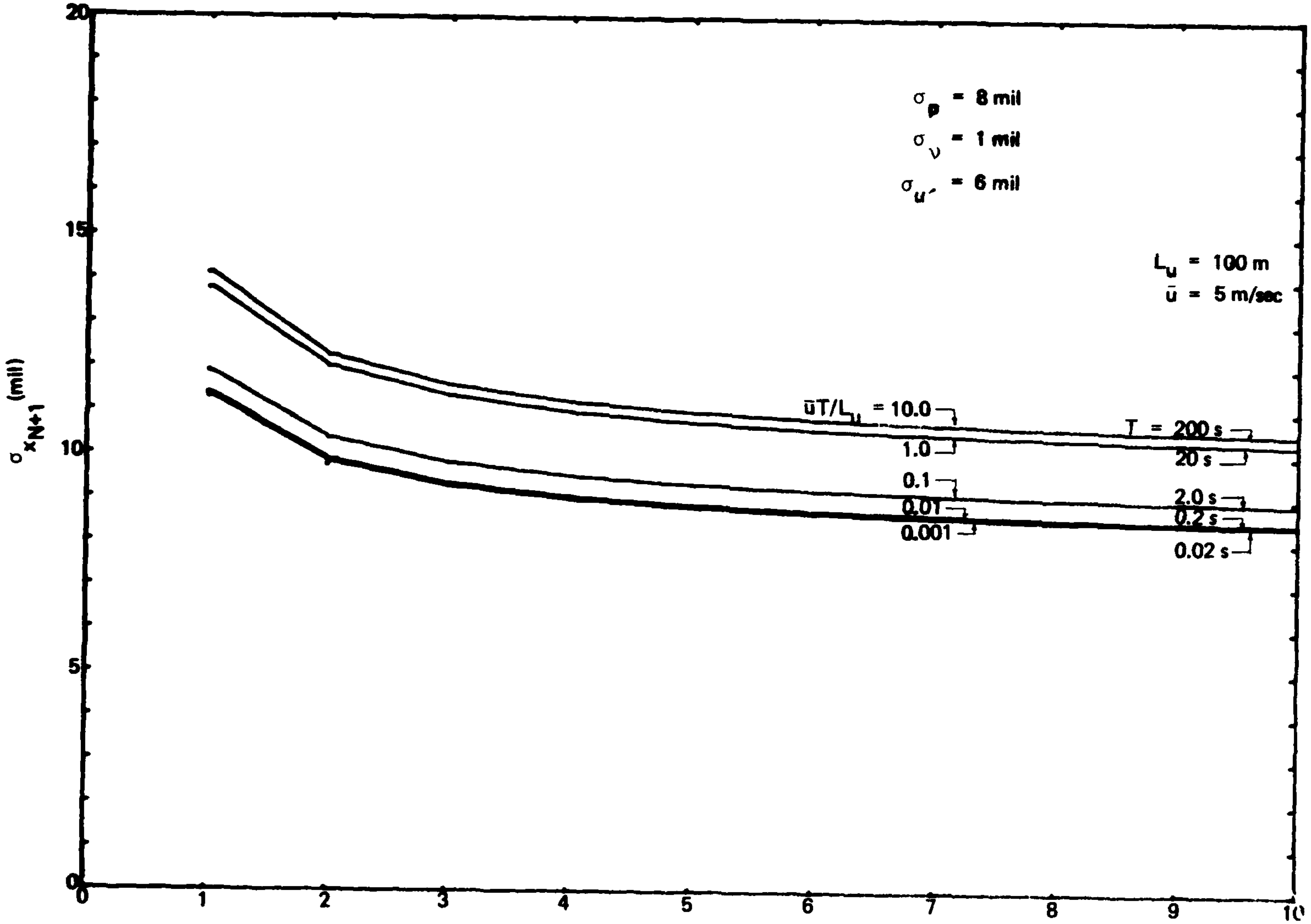

Figure 9. Number of rounds already fired.

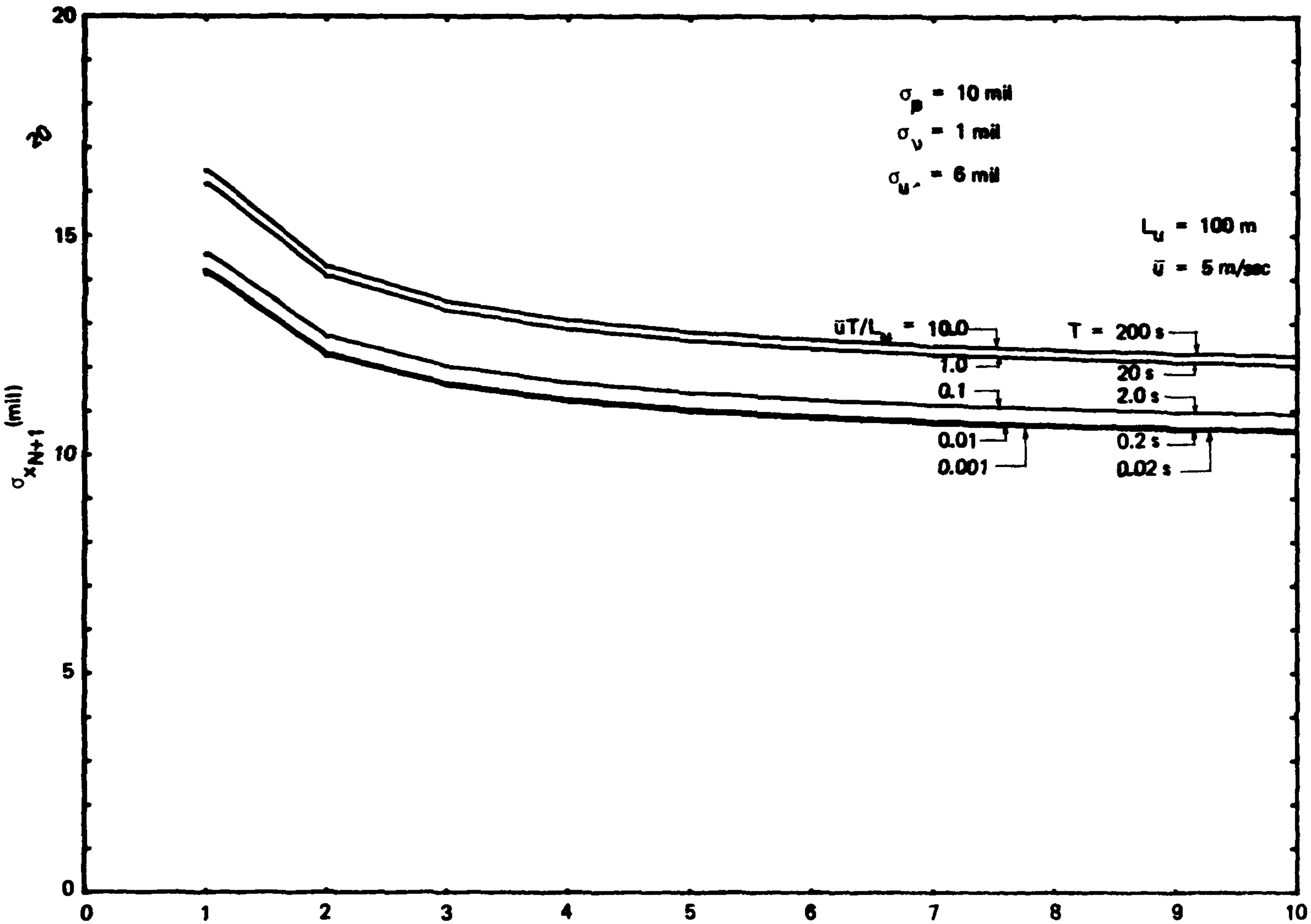

Figure 10. Number of rounds already fired.

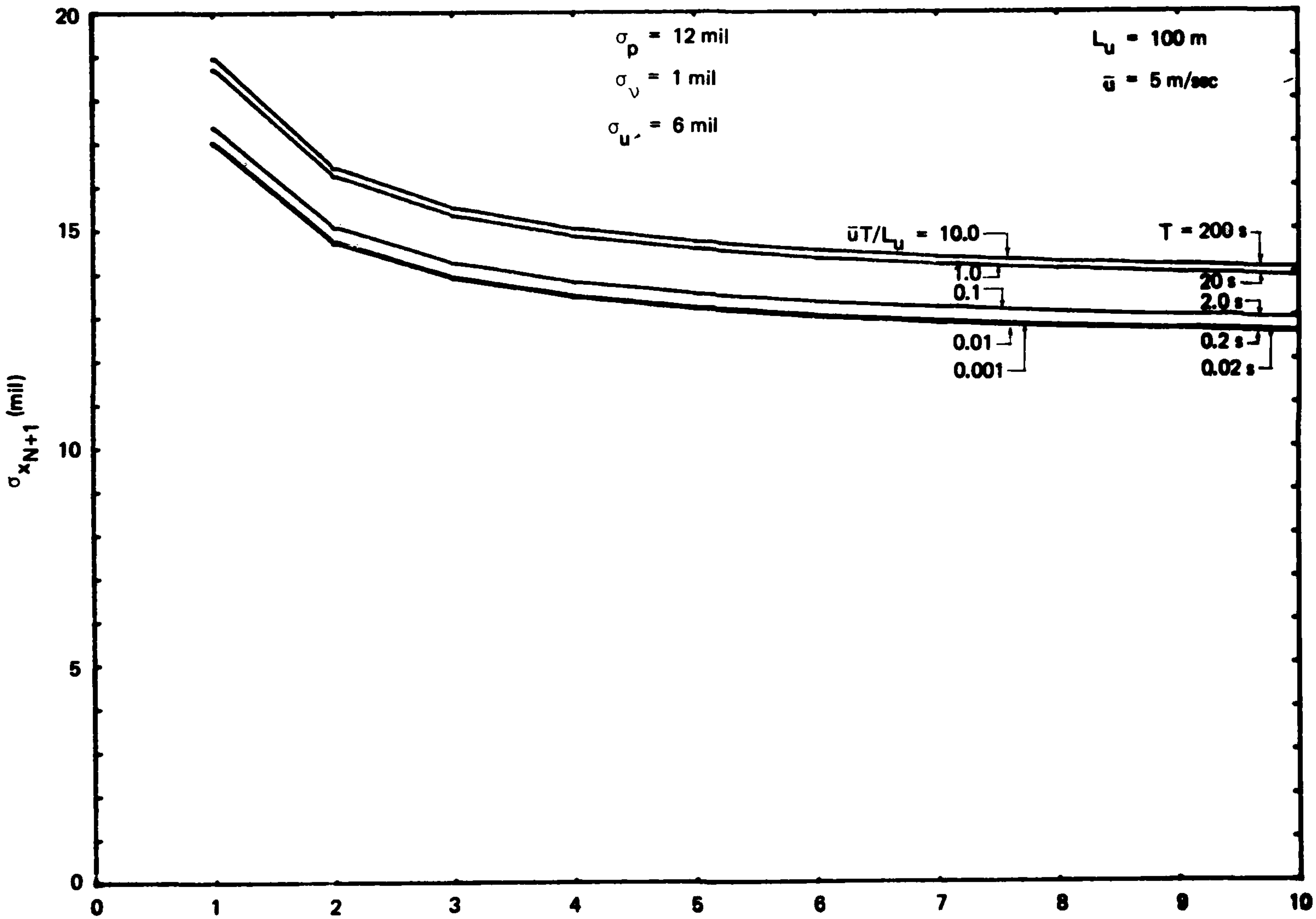

Figure 11. Number of rounds already fired.

IV. CONCLUSIONS AND RECOMMENDATIONS

Wetheru's Law of Suspended Judgement:

"Assumption is the mother of all screw-ups."

The results of incorporation of the revised wind (turbulence) model in the error analysis supports the overall conclusions of the previous studies.[1,2] Although it would appear that the time between rounds should be shorter, it does place those conclusions on a bit firmer foundation.

Probably the most significant contribution of this effort has been the incorporation of surface roughness via z_0 . Fichtl and McVehil[11] report a roughness of around 0.2 meter, where downwind of vegetation 1 to 2 meters in height, but 0.3 meter when 450 meters downwind, and 0.4 to 0.6 meter when 200 meters downwind of trees 10 to 15 meters tall. It would appear desirable not to fire from positions downwind of trees or buildings if the tactical situation permits. Deaves[12,13] has proposed a computational wind model which may be of use in analyzing the effects downwind of changes in surface roughness.

Of course, if the average is taken over too long a time one would have $\bar{u}$, not $\hat{u}(t+\tau)$. If the DAFR concept proceeds into development after demonstration, it is recommended that a probabilistic rather than a statistical filter be developed, that is, a Kalman filter[10] be utilized. The above turbulence model is suitable for Kalman filter formulation

$$\begin{pmatrix} \bar{u}_{k+1} \\ u'_{k+1} \end{pmatrix} = \begin{pmatrix} 1 , & 0 \\ 0 , & \rho(\tau) \end{pmatrix} \begin{pmatrix} \bar{u}_k \\ u'_k \end{pmatrix} + \begin{pmatrix} 0 \\ u''_k \end{pmatrix} \tag{30}$$

for the state equation, and

$$z_{k+1} = (1 \; , \; 1) \begin{pmatrix} \bar{u}_{k+1} \\ u'_{k+1} \end{pmatrix} + \nu_{k+1} \tag{31}$$

---

[11] G. H. Fichtl and G. E. McVehil, "Longitudinal and Lateral Spectra of Turbulence in the Atmospheric Boundary at the Kennedy Space Center," Journal of Applied Meteorology, Vol 9, February 1970, pp. 51-63.

[12] D. M. Deaves, "Computations of Wind Flow Over Changes in Surface Roughness," Journal of Wind Engineering and Industrial Aerodynamics, Vol 7, 1981, pp. 65-94.

[13] D. M. Deaves, "Terrain - Dependence of Longitudinal R.M.S. Velocities in the Neutral Atmosphere," Journal of Wind Engineering and Industrial Aerodynamics, Vol 8, 1981, pp. 259-275.

for the observation equation, where z_{k+1} is just u_{k+1} with observation noise, ν_{k+1}, added. Of course, these state and observation equations would have to be expanded to include the missile and tracker dynamics. Besides explicit incorporation of the Markov process in the filter, an error analysis would be generated in the filtering process.

"There is no law except the law that there is no law."

-John Archibald Wheeler
Physicist

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